PED Changes - Linear Demand Curve

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Definition of Price Elasticity of Demand (PED)

Price elasticity of demand (PED) measures the responsiveness of the quantity demanded of a good to a change in its price.

Formally:

$$\text{PED} = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}}$$

or, using calculus for a demand function $Q = f(P)$:

$$\text{PED} = \frac{dQ}{dP} \times \frac{P}{Q}$$

Key points:

- PED is usually **negative** for normal goods (because of the law of demand), but we often refer to its **absolute value** $|\text{PED}|$.
- Interpretation:
  - $|\text{PED}| > 1$: **Elastic** demand (quantity responds strongly to price changes).
  - $|\text{PED}| = 1$: **Unitary elastic** demand.
  - $|\text{PED}| < 1$: **Inelastic** demand (quantity responds weakly to price changes).

How PED Changes Along a Straight-Line Demand Curve

On a typical downward-sloping **straight-line (linear) demand curve**, PED is **not constant**:

- **High prices / low quantities (upper part of the curve)**: demand is **more elastic**.
- **Midpoint of the curve**: demand is **unitary elastic** ($|\text{PED}| = 1$).
- **Low prices / high quantities (lower part of the curve)**: demand is **more inelastic**.

Why PED changes even though the slope is constant

For a linear demand curve, the slope $\frac{dP}{dQ}$ (or $\frac{dQ}{dP}$) is constant, but PED depends on **both** the slope and the **ratio** $\frac{P}{Q}$:

$$\text{PED} = \frac{dQ}{dP} \times \frac{P}{Q}$$

- $\frac{dQ}{dP}$ is constant for a straight line.
- But $\frac{P}{Q}$ changes as you move along the curve.

So even with a constant slope, the **percentage** responsiveness changes because the base levels of P and Q change.

Intuitively:

- At the **top-left** (high P, low Q): a given absolute change in price is a **small percentage** change, but the same absolute change in quantity is a **large percentage** change → high elasticity.
- At the **bottom-right** (low P, high Q): the same absolute price change is a **large percentage** change, but the quantity change is a **small percentage** change → low elasticity.

Numerical Example

Consider a simple linear demand function:

$$P = 100 - 2Q \quad \text{or} \quad Q = 50 - 0.5P$$

Here, $\frac{dQ}{dP} = -0.5$ (constant).

PED at any point:

$$\text{PED} = \frac{dQ}{dP} \times \frac{P}{Q} = -0.5 \times \frac{P}{Q}$$

Point A: High price, low quantity (elastic region)

Let $Q = 10$:

- $P = 100 - 2(10) = 80$
- $\text{PED} = -0.5 \times \frac{80}{10} = -0.5 \times 8 = -4$

$|\text{PED}| = 4 > 1 \rightarrow$ **elastic** demand.

Point B: Midpoint (unitary elasticity)

Midpoint in quantity terms: $Q = 25$

- $P = 100 - 2(25) = 50$
- $\text{PED} = -0.5 \times \frac{50}{25} = -0.5 \times 2 = -1$

$|\text{PED}| = 1 \rightarrow$ **unitary elastic**.

Point C: Low price, high quantity (inelastic region)

Let $Q = 40$:

- $P = 100 - 2(40) = 20$
- $\text{PED} = -0.5 \times \frac{20}{40} = -0.5 \times 0.5 = -0.25$

$|\text{PED}| = 0.25 < 1 \rightarrow$ **inelastic** demand.

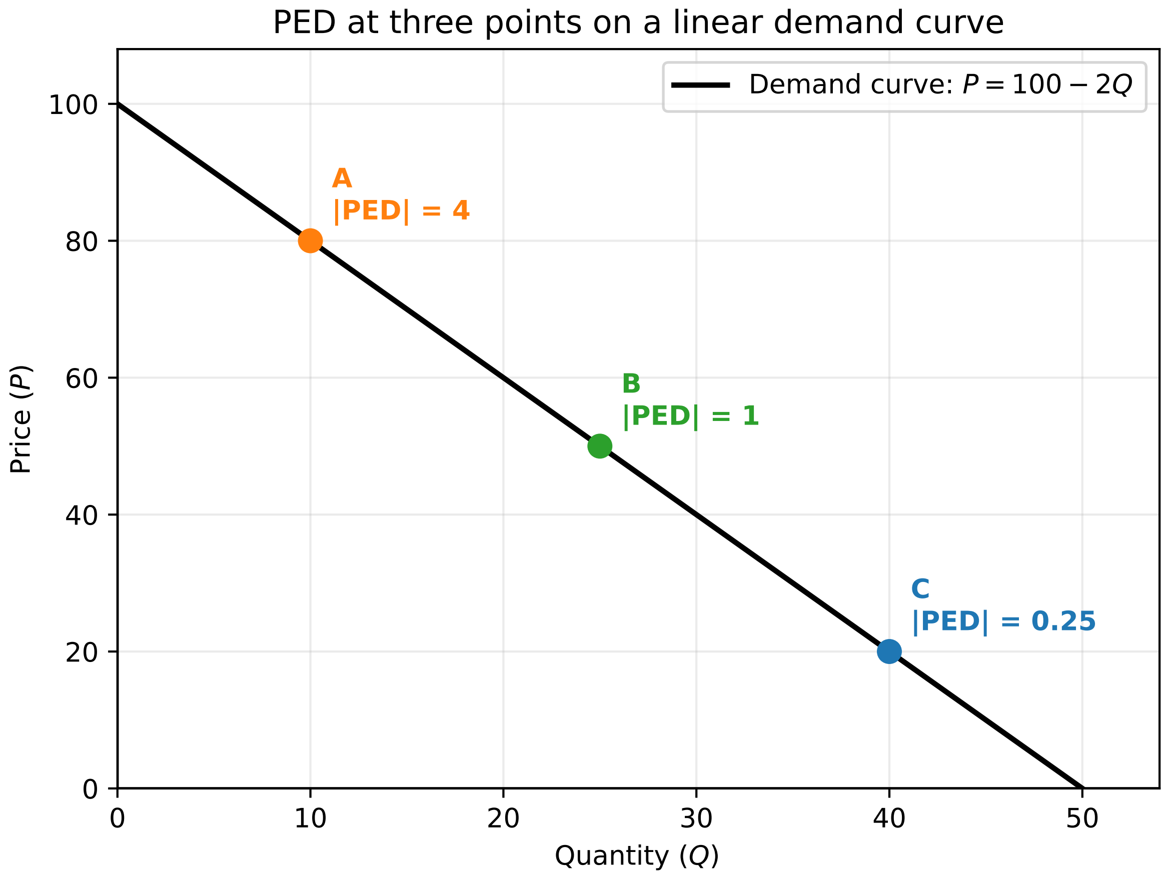

Figure 1: The numerical example: PED changes from elastic at A to unitary elastic at B and inelastic at C along the same straight-line demand curve.

So along the same straight line:

- **Top segment**: $|\text{PED}| > 1$ (elastic)
- **Midpoint**: $|\text{PED}| = 1$ (unitary)
- **Bottom segment**: $|\text{PED}| < 1$ (inelastic)

Explaining PED = 1 at the mid point

Another way of calculating PED is that on a demand curve,

PED = Lower Segment/Upper Segment

Let us see how this can be derived.

The Starting Point: The Elasticity Formula

The standard point elasticity of demand formula is:

$$\text{PED} = \frac{\%\Delta Q}{\%\Delta P} = \frac{dQ}{dP} \times \frac{P}{Q}$$

On a standard graph:

- Price (P) is on the vertical axis (Y-axis).
- Quantity (Q) is on the horizontal axis (X-axis).

Because the axes are flipped relative to typical math conventions (P is independent but on the Y-axis), the mathematical slope of the demand curve is $\frac{dP}{dQ}$. Therefore, $\frac{dQ}{dP}$ is actually $\frac{1}{\text{slope}}$ of the demand curve.

Setting up the Geometry

Imagine a straight-line demand curve that intersects the vertical Price axis at point A and the horizontal Quantity axis at point B. Let's pick any random point C on this demand curve to measure elasticity:

- Draw a horizontal line from C to the Price axis; call this point P_c (this represents the current Price).
- Draw a vertical line from C down to the Quantity axis; call this point Q_c (this represents the current Quantity).

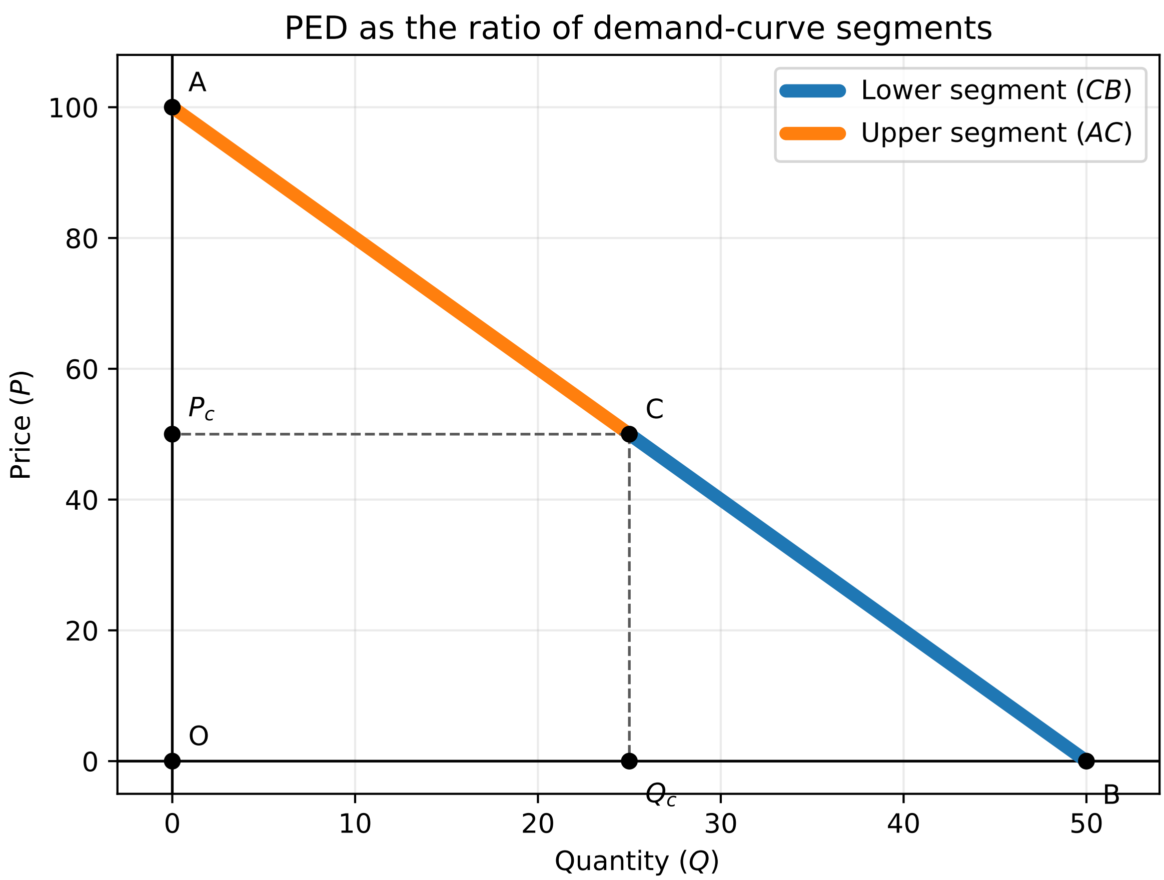

Figure 2: A straight-line demand curve showing the upper and lower segments used to derive PED at point C.

Notice the Similar Triangles

By dropping these lines, we create a series of similar triangles (triangles that have the exact same angles but different sizes):

1. The top triangle: $\triangle AP_cC$
2. The bottom triangle: $\triangle CQ_cB$

Because these triangles are similar, the ratios of their corresponding sides must be exactly equal.

Let's look at the components of our elasticity formula ($\frac{1}{\text{slope}} \times \frac{P}{Q}$):

- $\frac{1}{\text{slope}}$ (which is $\frac{\Delta Q}{\Delta P}$) can be represented by the base and height of the lower triangle: $\frac{Q_cB}{CQ_c}$.
- $\frac{P}{Q}$ is the price at point C divided by the quantity at point C, which matches the lengths $\frac{CQ_c}{OQ_c}$ (where O is the origin).

When you multiply them together:

$$\text{PED} = \left(\frac{Q_cB}{CQ_c}\right) \times \left(\frac{CQ_c}{OQ_c}\right)$$

Notice that CQ_c appears on both the top and the bottom, so they cancel each other out:

$$\text{PED} = \frac{Q_cB}{OQ_c}$$

Relating to the Hypotenuse (The Segments)

Because $\triangle AP_cC$ and $\triangle CQ_cB$ are similar, the ratio of their horizontal bases ($\frac{Q_cB}{OQ_c}$) is perfectly identical to the ratio of their hypotenuses along the demand curve itself.

- The line segment CB represents the Lower Segment (from point C down to the Quantity axis).
- The line segment AC represents the Upper Segment (from the Price axis down to point C).

Therefore, by geometric substitution:

$$\text{PED} = \frac{\text{Distance from } C \text{ to } B}{\text{Distance from } A \text{ to } C} = \frac{\text{Lower Segment}}{\text{Upper Segment}}$$

Visualizing the Result

This geometric approach gives us another way of thinking about elasticity along the demand curve.

- At the Price Intercept (Top): The lower segment is the entire line, and the upper segment is zero. $\text{PED} = \text{Length}/0 = \infty$ (Perfecty Elastic).
- At the Midpoint: The lower segment exactly equals the upper segment. $\text{PED} = 1/1 = 1$ (Unitary Elastic).
- At the Quantity Intercept (Bottom): The lower segment is zero, and the upper segment is the entire line. $\text{PED} = 0/\text{Length} = 0$ (Perfecty Inelastic).

Key Pattern to Remember (for Exams)

On a typical downward-sloping straight-line demand curve:

- **Upper half**: $|\text{PED}| > 1 \rightarrow$ elastic
- **Midpoint**: $|\text{PED}| = 1 \rightarrow$ unitary
- **Lower half**: $|\text{PED}| < 1 \rightarrow$ inelastic
- At the **price-axis intercept** (where $Q = 0$): $|\text{PED}| \rightarrow \infty$ (perfectly elastic in the limit)
- At the **quantity-axis intercept** (where $P = 0$): $\text{PED} = 0$ (perfectly inelastic in the limit)